EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

1. In the parabola $y^2 = 4ax$, the tangent at the point P, whose abscissa is equal to the latus ractum meets the axis in T & the normal at P cuts the parabola again in Q. Prove that PT : PQ = 4 : 5.

Sol.

2. Two tangents to the parabola $y^2 = 8x$ meet the tangent at its vertex in the points P & Q. If PQ = 4 units, prove that the locus of the point of the intersection of the two tangents is $y^2 = 8(x + 2)$.

Sol.

3. A variable chord t_1t_2 of the parabola $y^2 = 4ax$ subtends a right angle at a fixed point to of the curve. Show that it passes through a fixed point. Also find the co-ordinates of the fixed point.

Sol.

4. Two perpendicular straight lines through the focus of the parabola $y^2 = 4ax$ meet its directrix in T & T' respectively. Show that the tangents to the parabola parallel to the perpendicular lines intersect in the mid point of T T'.

Sol.

5. Two straight lines one being a tangent to $y^2 = 4ax$ and the other to $x^2 = 4by$ are at right angles. Find the locus of their point of intersection.

Sol.

6. A variable chord PQ of the parabola $y^2 = 4x$ is drawn parallel to the line y = x. If the parameters of the points P & Q on the parabola are p & q respectively, show that p + q = 2. Also show that the locus of the point of intersection of the normals at P & Q is 2x – y = 12.

Sol.

7. Show that an infinite number of triangles can be inscribed in either of the parabolas $y^2 = 4ax$ & $x^2 = 4by$ whose sides touch the other.

Sol.

8. If (x_1, y_1), (x_2, y_2) and (x_3, y_3) be three points on the parabola $y^2 = 4ax$ and the normals at these points meet in a point then prove that

$$\frac{x_1 - x_2}{y_3} + \frac{x_2 - x_3}{y_1} + \frac{x_3 - x_1}{y_2} = 0.$$

Sol.

9. Show that the normals at two suitable distinct real points on the parabola $y^2 = 4ax (a > 0)$ intersect at a point on the parabola whose abscissa > 8a.

Sol.

10. PC is the normal at P to the parabola $y^2 = 4ax$, C being on the axis. CP is produced out wards to Q so that PQ = CP; show that the locus of Q is a parabola, & that the locus of the intersection of the tangents at P & Q to the parabola on which they lie is $y^2 (x + 4a) + 16 a^3 = 0$.

Sol.

11. A quadrilateral is inscribed in a parabola $y^2 = 4ax$ and three of its sides pass through fixed points on the axis. Show that the fourth side also passes through fixed point on the axis of the parabola.

Sol.

12. Prove that the parabola $y^2 = 16x$ and the circle $x^2 + y^2 - 40x - 16y - 48 = 0$ meet at the point P(36, 24) and one other point Q. Prove that PQ is a diameter of the circle. Find Q.

Sol.

13. A variable tangent to the parabola $y^2 = 4ax$ meets the circle $x^2 + y^2 = r^2$ at P and Q. Prove that the locus of the mid point of PQ is $x(x^2 + y^2) + ay^2 = 0$.

Sol.

14. Show that the locus of the centroids of equilateral triangles inscribed in the parabola $y^2 = 4ax$ is the parabola $9y^2 - 4ax + 32a^2 = 0$.

Sol.

15. A fixed parabola $y^2 = 4ax$ touches a variable parabola. Find the equation to the locus of the vertex of the variable parabola. Assume that the two parabolas are equal and the axis of the variable parabola remains parallel to the x-axis.

Sol.

16. Show that the circle through three points the normals at which to the parabola $y^2 = 4ax$ are concurrent at the point (h, k) is $2(x^2 + y^2) - 2(h + 2a)x - ky = 0$

Sol.

17. Prove that the locus of the centre of the circle, which passes through the vertex of the parabola $y^2 = 4ax$ and through its intersection with a normal chord is $2y^2 = ax - a^2$.

Sol.

Read the information given and answer the questions 18-20.

Two equal parabolas P_1 and P_2 have their vertices at $V_1(0, 4)$ and $V_2(6, 0)$ respectively. P_1 and P_2 are tangent to each other and have vertical axes of symmetry.

18. Find the sum of the abscissa and ordinate of their point of contact.

Sol.

19. Find the length of latus rectum.

Sol.

20. Find the area of the region enclosed by P_1, P_2 and the x-axis.

Sol.